Although PDE method has limited application scenarios while solving financial problems compared with Time series analysis, proprietary and internal models, it is still a powerful and widely used method while pricing derivatives, especially an option. To price an option, the more common methods are closed-formula, Monte-Carlo simulation, PDE, lattice approach, etc. The relatively simple mathematical derivation of applying PDEs is one of advantages of solving option pricing problems in real world. Taking ordinary European options as an example, based on the no-arbitrage principle, the PDE of European options can be quickly deduced. Correspondingly, it is much more complicated to overthrow the BS formula. Also, the program of PDE numerical solution, such as Finite Difference Method, is relatively easy to implement. At the same time, it can be directly evaluated in terms of consistency, stability, convergence, etc. as we talked in class and the homework, which can improve the speed as much as possible on the premise of ensuring stability.

One of the PDE models that commonly used in financial engineering is Black-Scholes model or Black-Scholes-Merton PDE model, which is effective to estimate the price of an option. Financial engineers use this model to determine the initial capital required to perfectly hedge a short position in the option.

Black-Scholes PDEs for the price of an option is modeled as a geometric Brownian motion and we want it to hold for all x >= 0 and t \in[0,T]. We first analy Since it does not involve probability, which means we can solve it by PDE methods